Introduction to Nonlinear Dynamics
Indian Institute of Technology Madras
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# Selected Exercises 2

**Disclaimer:** Please note that this set of "Selected Exercises" contains additional problems for practise. However, unlike the assignment sheet, this should *not* be turned in! Also, this is not an exhaustive list of exercises. In fact, it is a subset of the exercises given in the book "Nonlinear Dynamics and Chaos", First Indian Edition (2007) by Steven H. Strogatz. Interested students are encouraged to solve all the exercise problems from the aforementioned book.

## 1 Existence and uniqueness

1) Consider the initial value problem $\dot{x} =\mid x \mid^{\frac{p}{q}}$, $x(0) = 0$, where $p$ and $q$ are positive integers with no common factors.
   a) Show that there are infinite number of solutions if $p < q$.
   b) Show that there is a unique solution if $p > q$.

2) Consider the equation $\dot{x} = rx + x^3$, where $r > 0$ is fixed. Show that $x(t) \to \pm\infty$ in finite time, starting from any initial condition $x_0 \neq 0$.

3) [*Infinitely many solutions with the same initial condition*] Show that the initial value problem $\dot{x} = x^{1/3}, x(0) = 0$, has an infinite number of solutions. (Hint: Construct a solution that stays at $x = 0$ until some arbitrary time $t_0$, after which it takes off.)

## 2 Potentials

1) For each of the following vector fields, plot the potential function $V(x)$ and identify all the equilibrium points and their stability.

   a) $\dot{x} = -\sinh x$

   b) $\dot{x} = 3$

2) [*Solutions to* $\dot{x} = f(x)$ *cannot oscillate*] Let $\dot{x} = f(x)$ be a vector field on the line. Use the existence of a potential function $V(x)$ to show that solutions $x(t)$ cannot oscillate.